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Roll No-12

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**Experiment No-07**

**Topic**- PAIRED HOTELLING’S T2 DISTRIBUTIONGENERALISATION OF STUDENTS t-TEST)

**Problem-** A group of 12 hypertension patients were administered 5 mg of the drug ‘Metoprolol’ . The drug is used to test high blood pressure and fast heart rate . The data on systolic blood pressure measured in mm/mg(X1) and heart rate is measured in ‘beats per minute’ (X2) of the patients before and 1 hour after administering the drug is given below .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Serial No.** | **Before administering the drug** | |  | |
| **BP**  **(1X1)** | **Heart rate (1X1)** | **BP**  **(2X1)** | **Heart rate (2X2)** |
| 1 | 160 | 88 | 155 | 83 |
| 2 | 180 | 99 | 162 | 85 |
| 3 | 130 | 80 | 125 | 70 |
| 4 | 175 | 92 | 165 | 89 |
| 5 | 140 | 83 | 137 | 80 |
| 6 | 110 | 77 | 103 | 75 |
| 7 | 155 | 85 | 150 | 81 |
| 8 | 145 | 83 | 132 | 80 |
| 9 | 135 | 81 | 130 | 73 |
| 10 | 117 | 78 | 112 | 71 |
| 11 | 169 | 90 | 160 | 85 |
| 12 | 150 | 82 | 147 | 78 |

Has the drug been able to significantly reduce the BP and heart rate on average ? Also find the 95% simultaneous confidence interval for the individual mean difference .

**Theory-**

Suppose (X1, X2,................. Xp)/ are measured at two stages as represented below –

1st stage**-**

|  |  |  |  |
| --- | --- | --- | --- |
| X1 | X2 | ..... | Xp |
| 1X11 | 1X21 | ..... | 1Xp1 |
| 1X12 | 1X22 | ..... | 1Xp2 |
|  |  |  |  |
| 1X1n | 1X2n | ..... | 1Xpn |

2st stage-

|  |  |  |  |
| --- | --- | --- | --- |
| X1 | X2 | ..... | Xp |
| 2X11 | 2X21 | ..... | 2Xp1 |
| 2X12 | 2X22 | ..... | 2Xp2 |
|  |  |  |  |
| 2X1n | 2X2n | ..... | 2Xpn |

The sample of differences is obtained as follows-

|  |  |  |  |
| --- | --- | --- | --- |
| X1 | X2 | ..... | Xp |
| 1X11-2X11=d11 | 1X21- 2X21=d21 | ..... | 1Xp1- 2Xp1=dp1 |
| 1X12 | 1X22-2X22=d22 | ..... | 1Xp2- 2Xp2=dp2 |
|  |  |  |  |
| 1X1n- 2X1n=d1n | 1X2n-2X2n=d2n | ..... | 1Xpn-2Xpn=dpn2Xpn |

Here the observed differences are

d̰j = (dj1,dj2,............djn)/ ; j=1,2,.......p

For a test of the hypothesis H0:D̰=0̰ (D is the population difference) against H1:D0̰ for a population , H0 is rejected at the level of significance α

if observed T2 > tabulated T2 .

Or, n >

Where is the upper (100α)thpercentile of the F distribution with p and (n-p) degrees of freedom

Here,

= and Sd = where, Sij=

Also, an 100(1-α)% simultaneous confidence interval for the individual mean difference Di are given by

di

Where diis the ith element of and is the ith diagonal element of Sd.

**Calculation-**

Here, the first stage of observation represents the values recorded before administering the drug. The second stage of observation represents the values recorded after administering the drug. The R-program for obtaining a solution of the given problem is as follows –

x11=c(160,180,130,175,140,110,155,145,135,117,169,150)

x12=c(88,99,80,92,83,77,85,83,81,78,90,82)

x21=c(155,162,125,165,137,103,150,132,130,112,160,147)

x22=c(83,85,70,89,80,75,81,80,73,71,85,78)

D1=x11-x21

D1

D2=x12-x22

D2

D1\_bar=mean(D1);D2\_bar=mean(D2)

D1\_bar

D\_bar=array(c(D1\_bar,D2\_bar),dim=c(2,1))

D\_bar

n=12

s11=var(D1)\*(n/(n-1))

s11

s12=cov(D1,D2)\*(n/(n-1))

s12

s21=s12

s21

s22=var(D2)\*(n/(n-1))

s22

Sd=array(c(s11,s12,s21,s22),dim=c(2,2))

Sd

p=2

t2\_cal=n\*t(D\_bar)%\*%solve(Sd)%\*%D\_bar

t2\_cal

t2\_tab=(((n-1)\*p)/(n-p))\*qf(0.95,p,n-p,0)

t2\_tab

LCL1=D\_bar[1]-(sqrt(t2\_tab)\*sqrt(Sd[1,1]/n))

LCL1

UCL1=D\_bar[1]+(sqrt(t2\_tab)\*sqrt(Sd[1,1]/n))

UCL1

LCL2=D\_bar[2]-(sqrt(t2\_tab)\*sqrt(Sd[2,2]/n))

LCL2

UCL2=D\_bar[2]+(sqrt(t2\_tab)\*sqrt(Sd[2,2]/n))

UCL2

**Conclusion-**

Since the calculated value of HOTELLING’S T2-statistic (i.e. 40.89267) is greater than the tabulated value vat 5% level of significance (i.e. 9.026206) , so we accept the null hypothesis and conclude that the drug has been able to significantly reduce the BP and Heart Rate on average

Also the 95% simultaneous confidence intervals for the individual mean differences are

|  |  |
| --- | --- |
| **Mean difference Di** | **95% confidence interval** |
| D1=7.333333 | (3.276116,11.39055) |
| D2=5.666667 | (2.473723,8.85961) |